MIDTERM AA214A – October 29, 2008
Instructor: Thomas H. Pulliam
25 Points

Instructions: Show all your work. Just writing the answer will not get you credit. If you get bogged down with some algebra, write out how you would proceed to complete the problem (it will get you some credit). The points for each problem are given along with hints and helpful information. Make sure you read the problem completely before proceeding.

1. Using Taylor Tables for

$$(\delta_x u)_j + \alpha(\delta_x u)_{j-1} = \frac{1}{\Delta x}\left(u_j + Au_{j-1} + Bu_{j-2}\right)$$

   (a) Find the values of α, A, B which result in a 2^{nd} order accurate method **(Points: 4)**

   (b) Find er_t **(Points: 2)**

   Hints:

$$u_{j+k} = u_j + (k\Delta x)\left(\frac{\partial u}{\partial x}\right)_j + \frac{1}{2}(k\Delta x)^2\left(\frac{\partial^2 u}{\partial x^2}\right)_j + \ldots + \frac{1}{n!}(k\Delta x)^n\left(\frac{\partial^n u}{\partial x^n}\right)_j + \ldots$$

2. (a) Find the Modified wave number expression (ik^*) for **(Points:2)**

$$(\delta_x u)_j = \frac{1}{4\Delta x}\left(-u_{j+2} + 4u_{j+1} - 4u_{j-1} + u_{j-2}\right)$$

   (b) Find the Modified wave number expression (ik^*) for the resulting scheme from problem 1. **(Points:2)**

   Note for both problems: DO NOT SIMPLIFY or TRY TO EXPAND IN A SERIES, EXPRESSIONS INVOLVING SIN's and COS's ARE ACCEPTABLE !!

Turn to Back

3. Consider the method

$$u_{n+1} = 4u_n - 3u_{n-1} - 2h(u')_{n-1}$$

applied to the representative equation

$$u' = \lambda u + ae^{\mu t}$$

(a) Identify the characteristic matrix $P(E)$ and particular $Q(E)$ operators as discussed in class **(Points:3)**

(b) Find the characteristic polynomial $P(\sigma)$ and the resulting σ **(Points:2)**

(c) Identify the principal and any spurious roots**(Points:1)**

(d) Find er_λ and identify the order of this method. **(Points:2)**

Help: $\sqrt{1 \pm \epsilon} = 1 \pm \frac{1}{2}\epsilon - \frac{1}{8}\epsilon^2 \pm \frac{1}{16}\epsilon^3 \cdots$

4. Consider the method

$$\begin{aligned} \tilde{u}_{n+\frac{1}{3}} &= u_n + \tfrac{h}{3}(u')_n \\ \widehat{u}_{n+\frac{1}{2}} &= u_n + \tfrac{h}{2}(\tilde{u}')_{n+\frac{1}{3}} \\ u_{n+1} &= u_n + h(\widehat{u}')_{n+\frac{1}{2}} \end{aligned}$$

applied to the representative equation $u' = \lambda u$

(a) Identify the characteristic matrix $[P(E)]$ operator as discussed in class **(Points:3)**

(b) Find and solve (for σ) the characteristic polynomial $P(\sigma)$ **(Points:2)**

(c) Find er_λ and identify the order of accuracy for this method. **(Points:2)**

Hints: use the vector made up of

$$\begin{bmatrix} \tilde{u}_{n+\frac{1}{3}} \\ \widehat{u}_{n+\frac{1}{2}} \\ u_n \end{bmatrix}$$

when forming the matrix operator